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Renormalization Group Fixed Points and the Higgs Boson Spectrum

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Abstract

We study numerically the renormalization group equations for the Higgs potential of the two doublet model assuming perturbative unification and sufficiently large initial quartic and Higgs-Yukawa couplings such that the full nonlinearities interplay. We obtain predictions for the physical Higgs boson spectrum in the two doublet model up to systematic differences in fermion coupling schemes. Unambiguous predictions emerge only when there exists a heavy generation in which quarks couple to both doublets. In other cases we find that the potential can become quartically unstable at low energies for arbitrary initial stable values of the coupling constants.

I. Introduction

If the breaking of the electroweak symmetries proceeds by way of elementary scalar fields and if one may presume perturbative unification at some large scale, M_X, then there are rather stringent conditions which must be met by the Higgs boson masses and fermion masses in various extensions of the standard model. These results are dictated by the renormalization group evolution of the various coupling constants in the theory. Furthermore, random initial values of coupling constants at M_X can be driven to universal low energy values at M_W. The Higgs bosons in multiple Higgs doublet generalizations of the standard model can have predictable masses up to the unknown vacuum expectation value ratios in the presence of heavy families of fermions. Moreover, even perturbatively weak initial values of the Higgs boson quartic coupling constants are driven to these fixed points by large fermion Yukawa couplings. Indeed, if one wants to "naturally" ensure a finite Higgs quartic interaction system for all scales below M_X (and random initial values) then we find that the theory only makes sense if there do exist heavy families.

We examine the two doublet model in detail. Here there is a further question: why does the two-doublet model lead to a breaking of the SU(2)xU(1) symmetry which maintains conservation of the electric charge? This is a statement about the vacuum expectation value alignment of the two doublets: they must go into a "ferromagnetic" alignment to maintain a residual $U(1)_{EM}$. Remarkably, in the cases in which the Higgs couplings are driven to fixed points we find that the ferromagnetic solution necessarily occurs. Hence, this approach offers a natural solution to the question of why electromagnetism is realised in the exact symmetry mode.

Unfortunately, without large Higgs-Yukawa couplings in the theory we can say almost nothing about the exact values of the Higgs boson masses. We find only that they tend to be small, i.e., less than 200 Gev in all cases and possibly accessible to the Tevatron. However, the stabilization of the evolution due to heavy fermions is compelling in itself so we have chosen to study this possibility in detail here. We shall also give a discussion of the less predictive cases in which no heavy fermions are involved.

There are familiar bounds on Higgs masses and Higgs-Yukawa coupling constants in the standard model[1-4]. Many of these follow strictly from the renormalization group and the assumption of the existence of a desert over which finiteness of couplings must be maintained up to some "unification" scale, M_X. Pendleton and Ross[2] first suggested that fermion masses and Higgs masses may be determined by the fixed points of the renormalization group equations. Subsequently, the physical nature of such fixed points was discussed and various implications were given[3], such as the mass predictions for a fourth generation and KM mixing angles[4]. These ideas have been further developed recently[5] and applied to model building in an interesting way[6] and are essential to a complete understanding of unified theories involving large initial coupling constants (by large couplings we do not mean the saturation of unitarity bounds, $g^2 \sim 4\pi$, but rather g^2 "of order unity").

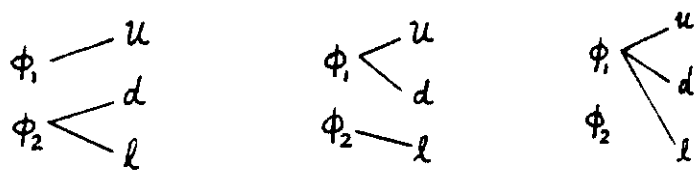
Our present study is essentially numerical, but we believe exhausts the list of qualitative phenomena which can occur in any natural multiple Higgs boson generalization of the standard model. Here we do not attempt to treat the evolution of the Lagrangian mass parameters of the Higgs fields. Since scalar field masses in the absence of chiral supermultiplets or as pseudogoldstone bosons are not multiplicatively renormalized, they are subject to large additive renormalizations of order M_X^2. Hence, the *renormalization group* logarithmic evolution is meaningless[7]. We opt presently to swap the unknown masses for unknown vacuum expectation value ratios and find general statements that are insensitive to the unknowns. A fixed point for us has a loose meaning; it is a physical result at low energies that is insensitive to initial conditions at high energies and reasonably insensitive to systematics.

II. *Renormalization Group Equations*

In this section we shall write down the one-loop renormalization group equations for a general class of models which are extensions of the Weinberg-Salam model to two complex Higgs doublets and N standard generations of fermions. We consider all possible coupling schemes of the fermions to the two Higgs doublets consistent with Glashow-Weinberg *naturalness*[8], i.e. *the natural avoidance of off diagonal neutral* couplings. Thus, we must ensure that no fermionic charge species couple to more than one Higgs doublet (but several fermionic charge species can couple to the same Higgs doublet). We furthermore assume throughout a negligible coupling to the neutrinos (hence our $N_g > 4$ analyses are strictly in some conflict with cosmological bounds, but ν's can be given finite yet small masses $\sim$ 5 Gev which evade these limits and are negligible in the present context).

Since we are free to interchange the definitions of ϕ_1 and ϕ_2 we need only consider distinguishable coupling schemes modulo this permutation symmetry. Furthermore, we have found empirically that the difference in the evolution of up and down quark flavors (generically) due to the g_1^2 coupling which breaks isospin is a net effect of order 2%. Therefore, to a good approximation we may assume a $u \leftrightarrow d$ permutation symmetry and consider coupling schemes which are distinct modulo this symmetry (this is not ordinary isospin; the mass matrix is diagonal and will in general break isospin via the Higgs-Yukawa couplings. But it is immaterial whether we call one quark "up" and another "down" up to 2% accuracy).

If we consider now a given generation and assume that the u-quarks, d-quarks and charged leptons all receive large masses through large Higgs-Yukawa couplings we arrive at the following three distinguishable coupling schemes:



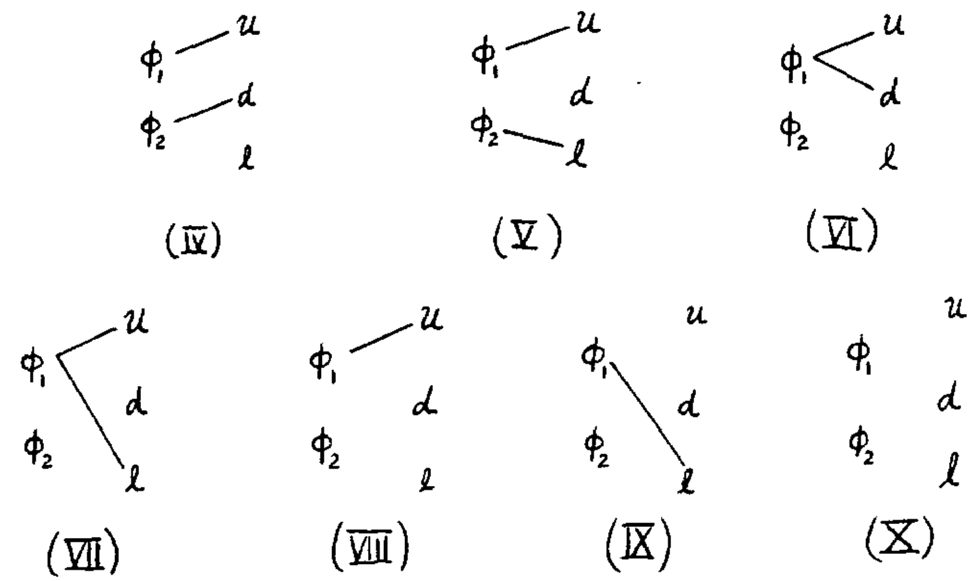
(I) (II) (III) (1)

(Our notation implies, e.g., that for case I the Higgs-Yukawa coupling is:

$$\bar{Q}_L \mathcal{U} u_R \phi_1^c + \bar{Q}_L \mathcal{D} d_R \phi_2 + \bar{L}_L \mathcal{L} e_R \phi_2 + h.c. \qquad (2)$$

where Q_L and L_L are, respectively, the left-handed quark and lepton doublets and u_R, d_R, and e_R are the right-handed up-quarks, down-quarks and charged leptons. We suppress generation indices. The $\mathcal{U}$, $\mathcal{D}$ and $\mathcal{L}$ are Yukawa coupling matrices.)

If we assume that there exist approximations in which certain or all of the fermionic charge species may have negligible masses then we arrive at the following additional cases:



(3)

The most general renormalizable and SU(2)xU(1) invariant potential with two Higgs doublets and reflection symmetries is well known(9) and is given in equation (A.1).

We now write down the renormalization group equations for a general model with parameters a_k and α_i that can take on different values for each of the above coupling schemes. The renormalization group equations for a 2 Higgs doublet model with the coupling scheme I has been given previously[10] and has been studied in ref.(11). We differ with the first of ref.(10) on the evolution equations of the $\mathcal{U}$ and $\mathcal{D}$ Yukawa matrices, and we further require a more general form as given below. In the following equations g_3, g_2, and g_1 represent the $SU(3)$, $SU(2)$ and $U(1)$ coupling constants respectively; $\mathcal{U}$, $\mathcal{D}$ and $\mathcal{L}$ represent the Yukawa matrices for the up type, down type and charged lepton species respectively. The λ_i ($i=1,..,5$) are the scalar quartic couplings in the potential of eq.(A.1). We let N=(number of generations), N_H=(number of Higgs doublets) and we define $D = 16\pi^2\mu\frac{\partial}{\partial\mu}$:

$$D g_3 = -n_3 g_3^3 \quad ; \quad n_3 = 11 - \frac{4}{3}N \tag{4a}$$

$$D g_2 = -n_2 g_2^3 \quad ; \quad n_2 = \frac{22}{3} - \frac{4}{3}N - \frac{1}{6}N_H \tag{4b}$$

$$D g_1 = +n_1 g_1^3 \quad ; \quad n_1 = \frac{20}{9}N + \frac{1}{6}N_H \tag{4c}$$

and

$$D\mathcal{L} = -\left(\frac{9}{4}g_2^2 + \frac{15}{4}g_1^2\right)\mathcal{L} + \frac{3}{2}\mathcal{L}\mathcal{L}^\dagger\mathcal{L}$$
$$+ \mathcal{L}\,\mathrm{Tr}\left(a_1\mathcal{L}\mathcal{L}^\dagger + 3a_2\mathcal{D}\mathcal{D}^\dagger + 3a_3\mathcal{U}\mathcal{U}^\dagger\right) \tag{5a}$$

$$\mathcal{D}\mathscr{D} = -\left(8g_3^2 + \frac{9}{4}g_2^2 + \frac{5}{12}g_1^2\right)\mathscr{D} + \frac{3}{2}\mathscr{D}\mathscr{D}^\dagger\mathscr{D} + a_4\,\mathcal{U}\mathcal{U}^\dagger\mathscr{D}$$
$$+ \mathscr{D}\,\mathrm{Tr}\left(a_5\mathcal{L}\mathcal{L}^\dagger + 3a_6\mathscr{D}\mathscr{D}^\dagger + 3a_7\mathcal{U}\mathcal{U}^\dagger\right) \qquad (5b)$$

$$\mathcal{D}\mathcal{U} = -\left(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}g_1^2\right)\mathcal{U} + \frac{3}{2}\mathcal{U}\mathcal{U}^\dagger\mathcal{U} + a_8\,\mathscr{D}\mathscr{D}^\dagger\mathcal{U}$$
$$+ \mathcal{U}\,\mathrm{Tr}\left(a_9\mathcal{L}\mathcal{L}^\dagger + 3a_{10}\mathscr{D}\mathscr{D}^\dagger + 3a_{11}\mathcal{U}\mathcal{U}^\dagger\right) \qquad (5c)$$

and

$$\mathcal{D}\lambda_1 = 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2$$
$$-3\lambda_1\left(3g_2^2 + g_1^2\right) + \frac{3}{2}g_2^4 + \frac{3}{4}\left(g_2^2 + g_1^2\right)^2$$
$$+4\lambda_1\,\mathrm{Tr}\left(\alpha_1\mathcal{L}\mathcal{L}^\dagger + 3\alpha_2\mathscr{D}\mathscr{D}^\dagger + 3\alpha_3\mathcal{U}\mathcal{U}^\dagger\right)$$
$$-4\,\mathrm{Tr}\left[\alpha_4(\mathcal{L}\mathcal{L}^\dagger)^2 + 3\alpha_5(\mathscr{D}\mathscr{D}^\dagger)^2 + 3\alpha_6(\mathcal{U}\mathcal{U}^\dagger)^2\right] \qquad (6a)$$

$$\mathcal{D}\lambda_2 = 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2$$
$$-3\lambda_2\left(3g_2^2 + g_1^2\right) + \frac{3}{2}g_2^4 + \frac{3}{4}\left(g_2^2 + g_1^2\right)^2$$
$$+4\lambda_2\,\mathrm{Tr}\left(\alpha_7\mathcal{L}\mathcal{L}^\dagger + 3\alpha_8\mathscr{D}\mathscr{D}^\dagger + 3\alpha_9\mathcal{U}\mathcal{U}^\dagger\right)$$
$$-4\,\mathrm{Tr}\left[\alpha_{10}(\mathcal{L}\mathcal{L}^\dagger)^2 + 3\alpha_{11}(\mathscr{D}\mathscr{D}^\dagger)^2 + 3\alpha_{12}(\mathcal{U}\mathcal{U}^\dagger)^2\right] \qquad (6b)$$

$$\mathcal{D}\lambda_3 = (\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2$$
$$-3\lambda_3\left(3g_2^2 + g_1^2\right) + \frac{9}{4}g_2^4 + \frac{3}{4}g_1^4 - \frac{3}{2}g_2^2g_1^2$$
$$+2\lambda_3\,\mathrm{Tr}\left(\alpha_{13}\mathcal{L}\mathcal{L}^\dagger + 3\alpha_{14}\mathscr{D}\mathscr{D}^\dagger + 3\alpha_{15}\mathcal{U}\mathcal{U}^\dagger\right)$$
$$-12\alpha_{16}\,\mathrm{Tr}\left(\mathscr{D}\mathscr{D}^\dagger\mathcal{U}\mathcal{U}^\dagger\right) \qquad (6c)$$

$$\begin{aligned} D\lambda_4 = {} & 2(\lambda_1+\lambda_2)\lambda_4 + 4(2\lambda_3+\lambda_4)\lambda_4 + 8\lambda_5^2 \\ & - 3\lambda_4(3g_2^2+g_1^2) + 3g_2^2 g_1^2 \\ & + 2\lambda_4 \operatorname{Tr}(\alpha_{17}\mathcal{L}\mathcal{L}^\dagger + 3\alpha_{18}\mathcal{D}\mathcal{D}^\dagger + 3\alpha_{19}\mathcal{U}\mathcal{U}^\dagger) \\ & + 12\alpha_{20}\operatorname{Tr}(\mathcal{D}\mathcal{D}^\dagger\mathcal{U}\mathcal{U}^\dagger) \end{aligned} \tag{6d}$$

$$\begin{aligned} D\lambda_5 = {} & \lambda_5 \big[2(\lambda_1+\lambda_2) + 8\lambda_3 + 12\lambda_4 - 3(3g_2^2+g_1^2) \\ & + 2\operatorname{Tr}(\alpha_{21}\mathcal{L}\mathcal{L}^\dagger + 3\alpha_{22}\mathcal{D}\mathcal{D}^\dagger + 3\alpha_{23}\mathcal{U}\mathcal{U}^\dagger) \big] \end{aligned} \tag{6e}$$

The values of n_1, n_2 and n_3 in eq.(4) are given in Table I for the different cases we shall consider. The values of a_i and α_j depend upon the coupling schemes and are given in Table II and Table III.

The standard one Higgs doublet case is simply obtained by setting λ_2, λ_3, λ_4, and λ_5 to zero and choosing the appropriate values of n_1 through n_3 and the a_i in the first columns of Tables I and II. The appropriate renormalization group equations for each of the 10 coupling schemes in the 2 Higgs doublet case can be read off of the Tables.

III. Discussion of Results

In this section we discuss the results of our numerical integration of the renormalization group equations. Our focus is on the scalar quartic couplings which determine the Higgs boson spectrum. In part A we discuss the results for the standard one doublet model. Part B contains the results of our study of two-Higgs doublet models. We shall see that the behavior of the 10 distinct coupling schemes can be divided into three classes of general results: (1) "non-leptonic" coupling, (2) "semi-leptonic" coupling and (3) "leptonic" coupling, depending upon how the doublets are coupled to the heavy fermions. We note that only class (1) exhibits fixed point behavior while class (3) can have diverging couplings at low energies.

In this analysis we integrate the renormalization group equations from M_X, which is chosen to be 10^{15}Gev throughout (a larger M_X, say 10^{17}Gev, will not significantly affect these results), down to a weak interaction scale of 100 Gev. For simplicity we assume that the Cabibbo-KM coupling matrices are diagonal. This is reasonable in that in the large coupling limits these matrix elements are generally driven diagonal rapidly by the renormalization group evolution[(4)]. Values of the gauge couplings can be derived from their measured values at low energies. We take: $g_3=.56*g_2$; $g_1=.43$, evaluated at M_X.

The light fermions ($m < m_{top}$) have negligible effects on the evolution of the coupling constants, except for the counting in the usual gauge coupling beta-functions. We shall simply set the Higgs-Yukawa coupling constants of the light fermions to zero.

Fixed points of the scalar quartic couplings are universal values attained from a sample of random initial values. These fixed points will in general depend upon the coupling scheme employed and the number of heavy generations assumed; within a given scheme they are remarkably well-defined. These are physical, quasi-fixed points in the sense of reference (3).

It is important that the λ_i at M_X respect the stability conditions of eq.(A.4). If these do not hold then there is a direction in group space in which $\langle\phi_1\rangle$ and $\langle\phi_2\rangle$ can grow to decrease the vacuum energy without bound.

This raises an interesting question: If we start with the λ_i respecting eq.(A.4) at M_X and evolve down in energy can we ever reach a scale M' at which the λ_i fail to respect the stability conditions of eq.(A.4)? If so, then the scale M' must be associated with the scale of a symmetry breaking. Presumably at M' the $\langle\phi_i\rangle$ will begin to grow without bound until $\langle\phi_i\rangle \sim M'$. This presumably drives the effective mass scale of the λ_i above M' and the stability is recovered. Hence, this must correspond to a stable minimum with $\langle\phi_i\rangle \sim M'$.

This mechanism is not equivalent to Coleman-Weinberg symmetry breaking and can be strongly first order since the quadratic terms could define a stable minimum (it seems akin to the idea of "quantum resuscitation" of Dimopoulos and Georgi [12]). We shall find that this sort of phenomenon does occur in the leptonic coupling schemes described below.

(A) The Standard One Doublet Model

Consider the standard model with three generations and a possible arbitrarily heavy t-quark. The vacuum expectation value of the neutral Higgs boson is determined, and thus given the quartic coupling the mass of the physical Higgs scalar is determined.

In Fig.(1) we give the simultaneous evolution of the scalar quartic coupling, λ , and the t-quark Yukawa coupling, g_t, for arbitrary initial values. We see that for sufficiently large initial values of λ and g_t that they simultaneously approach a fixed point. For smaller initial g_t the evolution terminates on the dashed line of Fig.(1). This can be turned into a relationship between the mass of the t-quark and that of the neutral scalar, which is presented in Fig.(2). Curiously, for a large range of m_t we see that m_H lies around 170 Gev. We note that if we allow for small λ the criterion that it should not be negative between M_W and M_X gives a lower bound on m_H (see for example ref.(13)).

There have been previous analyses of the Higgs boson mass involving the renormalization group and many attempts to place upper bounds on the mass of the Higgs scalar[1,13]. Pendleton and Ross first suggested that the mass might be determined by infra-red fixed points of the renormalization group equations. However, we disagree with their prediction by a factor of approximately two since they assume the fixed point is the exact mathematical one, which is never reached by the decoupling scale of ~ 100 Gev. Another approach consists of integrating the renormalization group equations from M_W to M_X and upper (lower) bounds on λ follow by requiring that it not be singular (negative) over the range of the desert[1]. The upper bounds will generally occur at the fixed point of the evolution.

If there is a heavy fourth generation, the mass of the Higgs boson increases. For example, m_H lies between 200 and 220 Gev for m_t between 30 and 100 Gev in the presence of a heavy fourth generation. Contrary to the three generation case, m_H decreases with increasing m_t. However, m_H increases with the mass of the heaviest fermion. This can be seen from the renormalization group equation of λ, for the fixed point of λ increases if the Yukawa coupling of the heavy fermion is increased.

(B) Two Higgs Doublet Models

The qualitative behavior of the renormalization group evolution of the quartic couplings can be divided into three classes:

(1) "Non-leptonic" coupling (schemes I and IV). Each doublet is coupled to at least one heavy quark.

(2) "Semi-leptonic" coupling (schemes II and V). One doublet is coupled to at least one heavy lepton (but no quarks) and the other doublet is coupled to at least one heavy quark.

(3) "Leptonic" coupling (schemes III, VI, VII, VIII, IX and X). One or both Higgs doublets are not coupled to any heavy fermions.

Our conclusions presently are based upon the study of the two Higgs doublet model, but we believe that this qualitative classification will hold for any further generalization of the model. We note that two doublets saturate the "non-leptonic" schemes if natural off diagonal neutral coupling suppression is assumed. With three Higgs doublets one saturates the "semi-leptonic" schemes. Any further generalization will necessarily have doublets coupled to light objects (neutrinos) or no coupling to fermions at all and the "leptonic" mode will be realized.

Fixed points of the scalar quartic couplings, and hence definite predictions for the Higgs masses, are obtained only for the "non-leptonic" schemes. The case with only three standard generations and no heavy fourth generation corresponds either to scheme VIII or X, depending upon how heavy the t-quark is. Thus, our results indicate that fixed point behavior will not occur in this case.

We can understand why heavy quarks are necessary for the quartic couplings to reach fixed points by examining the structure of the renormalization group equations. Consider the extreme case in which no heavy fermion is coupled to either Higgs doublet (X). Consider then the renormalization group equation for λ_1 (similarly λ_2). It can be written as:

$$\mathcal{D}\lambda_1 = 12\left[\lambda_1 - \tfrac{1}{8}(3g_2^2+g_1^2)\right]^2 + 2\lambda_3^2 + 2(\lambda_3+\lambda_4)^2 + 2\lambda_5^2 + \tfrac{9}{16}(g_2^4+g_1^4) + \tfrac{3}{8}g_2^2 g_1^2 \qquad (7)$$

Note that the rhs is always positive and thus no fixed point exists (zero is not a fixed point because the rhs is not proportional to λ_1). As a result, λ_1 (or λ_2) always decreases from its initial value at M_X and can eventually exit the stability region becoming negative and eventually negative infinite. Coupling ϕ_1 (or ϕ_2) to heavy fermions introduces the negative stabilizing term:

$$- 4\,\mathrm{Tr}\left[\alpha_4(\mathcal{L}\mathcal{L}^\dagger)^2 + 3\alpha_5(\mathcal{D}\mathcal{D}^\dagger)^2 + 3\alpha_6(\mathcal{U}\mathcal{U}^\dagger)^2\right]$$

to the renormalization group equation, where at least one of the α's is nonvanishing. In this case a fixed point can exist and be reached if the Yukawa couplings are sufficiently large. It is also clear that a heavy quark is more effective than a heavy lepton in driving the system toward a fixed point because of the color factor of three and the larger fixed point values of the quark Yukawa couplings.

It is somewhat remarkable that a system of Yukawa and quartic couplings can give rise to a pseudo-asymptotically free behavior at low energies. Of course, if the system is evolved above M_X we would see all couplings diverging eventually, but at low energies as one or more couplings diverge they can drag the others along. Thus, with the theory defined at M_X to be finite we have the possibility of the theory becoming non-perturbatively strong at the weak scale by these effects. This is reminiscent of technicolor and might be applied to generate the breaking of the weak interaction symmetries at low energies.

In the semi-leptonic schemes the important stabilizing term, $12\,\mathrm{Tr}(\mathcal{D}\mathcal{D}^\dagger\mathcal{U}\mathcal{U}^\dagger)$, is absent from the rhs of the renormalization group equations for λ_3 and λ_4. Consequently these couplings do not reach fixed points although λ_1 and λ_2 do.

We now discuss our results in the three classes of coupling schemes.

(1) Nonleptonic Coupling (I and IV)

The flow of the λ_i toward fixed points is shown in Figures (3a,b and c) for coupling scheme I with N=4. The rate of approach is indicated by the solid circles; the distance between solid circles is one tenth of the logarithmic energy range between 10^{15}Gev and 100 Gev. The fixed points are reached by about 80% of the running time from M_X, i.e., at a scale of 10 to 100 Tev, independent of initial values. Each of the λ_i reach fixed points with the exception of λ_5.

As is evident from eq.(6e) the fixed point for λ_5 is zero. This fixed point corresponds to the Peccei-Quinn U(1) symmetry. However, λ_5 never really attains zero in the finite running time and becomes sensitive to initial values.

It is interesting to note in Fig.(3c) that λ_4 reaches its fixed point (which is negative) for arbitrary initial values. Negative λ_4 is essential for an electric charge conserving vacuum in which the two doublets are aligned in their vacuum expectation values (for positive λ_4 the vacuum energy is minimized by the counter-aligned configuration which corresponds to a broken electric charge and thus a massive photon). It is somewhat remarkable that the renormalization group fixed point for the two Higgs doublet model with a heavy fourth generation will select the physically interesting vacuum!

Our numerical results for the fixed points of the λ_i are summarized in Tables IV and V for coupling schemes I and IV. We have studied the cases of one and two heavy generations. For simplicity we take the Higgs-Yukawa couplings of the heavy generations to be equal at M_X. Varying the initial values of the Yukawa couplings will modify the results by about 10% (assuming always large, i.e., greater than unity, initial values).

The fixed points are approached from a large sample of initial values of the λ_i at M_X. The errors quoted in the tables reflect the fact that not every set of initial values of the λ's attain the same fixed point in the finite running time from M_X. Wherever these errors are not reported it means that they are negligible. These errors are generally rather small. Clearly λ_5 has a large relative error because it has a fixed point of zero, yet in absolute magnitude this error is comparable to that of λ_4.

The renormalization group equations respect the symmetry $\lambda_5 \to -\lambda_5$ and the sign of λ_5 can never change during the evolution. Hence we quote only the results for $|\lambda_5|$.

We have shown how the fixed points vary with the mass of the t-quark. In each case it is seen that there is essentially no difference between m_t=0 and m_t=50 Gev, whereas a heavier t-quark tends to lower the magnitudes as well as the errors of the fixed points. Similar effects occur with the addition of extra heavy generations. Comparing Table IV with Table V we see that the fixed point for λ_1 is about the same for both coupling schemes I and IV since ϕ_1 is coupled to the same heavy u-quarks in both cases. The fixed point for λ_2 and consequently those for λ_3, λ_4 and λ_5 are smaller in magnitude in scheme I because ϕ_2 is coupled to heavy leptons in addition to d-quarks.

The masses of the physical Higgs bosons can be obtained from the fixed points by eq.(A.6a through d). These are displayed in Tables VI and VII. The masses of the neutral scalars depend upon the unknown ratio v_2/v_1 of the vacuum expectation values of the two Higgs doublets.

They are plotted as a function of this ratio in Fig.(4a and b) for coupling schemes I and IV. It is seen that these masses vary in a finite range as the vacuum expectation value ratio runs from 0 to ∞. Since λ_5 never reaches its fixed point we cannot quote precise predictions for the neutral pseudoscalar; the values quoted in Tables VI and VII are obtained from the "mean values" of λ_5 in Tables IV and V. It is evident, however, that this mass tends to be small.

All of these masses lie in an interesting range which is accessible to experiment in the near future.

(2) *Semi-leptonic Coupling (II and V)*

In both these schemes ϕ_2 is coupled only to leptons. Qualitatively we find that λ_1 approaches a stable fixed point while λ_2 has a larger variation in its value at 100 Gev (this variation is controlled by the magnitude of the lepton Yukawa couplings and can be made small for large Yukawa couplings at M_X). λ_3, λ_4, and λ_5 are driven small, but do not reach fixed points. Just as in the non-leptonic case λ_4 can be driven negative from a positive initial value provided the Yukawa couplings are sufficiently large.

We find that if we start with initial λ_i satisfying the stability conditions, eq.(A.4), then λ_3, λ_4, and λ_5 are always small ($\lesssim .1$) at the weak scale. This can be understood from the renormalization group equations. In both of these schemes α_{16} and α_{20} are zero. Hence, neglecting contributions of the form (gauge coupling)^4, which is always small, we see that the system of equations may be written:

$$\mathcal{D}\begin{pmatrix}\lambda_3\\ \lambda_4\\ \lambda_5\end{pmatrix} = A(\lambda_i;\, g_2, g_1;\, \mathcal{L}, \mathcal{D}, \mathcal{U})\begin{pmatrix}\lambda_3\\ \lambda_4\\ \lambda_5\end{pmatrix} \tag{8}$$

which has a fixed point at $\lambda_3 = \lambda_4 = \lambda_5 = 0$. Hence, in the "semi-leptonic" scheme these parameters behave like λ_5 in the preceding case.

The masses of the neutral scalars can be accurately computed in terms of v_2/v_1 since m_1 and m_2 are insensitive to λ_3, λ_4, and λ_5 provided they are small. The results are similar to those presented in Fig.(4). The ranges of m_1 and m_2 for each of the cases considered is presented in Table VIII.

Since λ_4 and λ_5 do not attain fixed points we cannot predict the masses of the charged scalar or the neutral pseudoscalar. However, they are small in this case and we typically find $m_p \sim m_\pm \sim 50$Gev. for $\lambda_4, \lambda_5 \sim .04$.

Again, the masses are in interesting ranges though are not controlled by their fixed points to the extent they are in the nonleptonic schemes.

(3) Leptonic Coupling (III,VI,VII,VIII,IX,X)

Presently one or both of the Higgs doublets are coupled to fermions with negligible Higgs-Yukawa couplings. Unlike the previous two cases λ_4 cannot be driven negative at low energies if it is positive at M_X. In case X in which both ϕ_1 and ϕ_2 are not coupled to heavy fermions we find that at some scale λ_4 (taken to be initially negative) starts to increase in magnitude. When λ_4 is sufficiently large in magnitude it causes λ_1 and λ_2 to become negative and the system becomes unstable. The energy at which this occurs depends upon the initial values in a sensitive way.

In the other cases in which only ϕ_1 is coupled to heavy fermions we find that λ_1 reaches its fixed point but λ_2, λ_3, λ_4, and λ_5 do not. As in the semi-leptonic case λ_3, λ_4 and λ_5 tend to be small. The system will not become unstable as in case X provided the fermions are sufficiently heavy. The resulting masses of the Higgs bosons are not determined in these cases.

Here we encounter another phenomenon. It is possible to find initial configurations of the λ_i at M_X satisfying eq.(A.4) which descend to lower energy scales at which eq.(A.4) become violated. A typical initial configuration is evolved in Fig.(5).

Here we see that λ_1, λ_2 and λ_3 monotonically decrease while the negative λ_4 increases and eventually turns over to decrease again. λ_5 if initially positive monotonically increases. We quickly reach an energy scale at which eq.(A.4a) is no longer satisfied. Here we should presumably halt the evolution since the symmetry breaking will be associated with this scale. If, however, we continue the evolution we eventually reach a scale at which either λ_1 or λ_2 become negative, and subsequently the couplings blow up.

We have not found cases in which the couplings begin at M_X satisfying eq.(A.4), becoming arbitrarily large at low energies while always satisfying the stability conditions of eq.(A.4). We therefore conjecture that the theory always becomes quartically unstable before becoming a strongly interacting theory.

One can presumably tune the initial values at M_X so that the cross-over to quartic instability occurs at M_W. The theory would still be perturbative at that scale and the electroweak interactions become broken for massless scalars. Such a mechanism might provide an alternative to technicolor schemes, but seems somewhat contrived to us.

Appendix

We discuss in this appendix the properties of the two Higgs doublet potential. Though much of this has been discussed previously[9] we include it here for convenience.

The most general renormalizable, SU(2)xU(1) invariant, and reflection invariant potential is:

$$V(\phi_1,\phi_2) = \mu_1^2\phi_1^\dagger\phi_1 + \mu_2^2\phi_2^\dagger\phi_2 + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4|\phi_1^\dagger\phi_2|^2 + \frac{\lambda_5}{2}\left[(\phi_1^\dagger\phi_2)^2 + h.c.\right] \tag{A.1}$$

where $\phi_1=(\phi_1^+,\phi_1^0)$ and $\phi_2=(\phi_2^+,\phi_2^0)$ denote the two scalar doublets with weak hypercharge +1. It is necessary to impose the discrete symmetry $\phi_1 \longrightarrow -\phi_1$ and $\phi_2 \longrightarrow -\phi_2$ (together with appropriate transformations for the fermions depending upon the form of the Higgs-Yukawa interactions) in order to avoid flavor-changing neutral couplings.

All coupling constants are real. More generally, the last term in $V(\phi_1,\phi_2)$ can be written as:

$$\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 \tag{A.2}$$

However, the phase of λ_5 can be absorbed into the relative phase between ϕ_1 and ϕ_2. Consequently, λ_5 can be chosen real without loss of generality. λ_5 explicitly breaks a Peccei-Quinn U(1) symmetry giving a mass to the pseodoscalar Higgs boson.

The requirement that the the potential energy of the vacuum be bounded below necessarily implies the following conditions in tree-approximation:

$$\lambda_1 > 0$$
$$\lambda_2 > 0 \tag{A.3}$$

and:

$$\sqrt{\lambda_1\lambda_2} > -\lambda_3 + |\lambda_4| + |\lambda_5| \qquad \text{if} \quad \lambda_4 < 0 \tag{A.4a}$$

$$\sqrt{\lambda_1\lambda_2} > -\lambda_3 - \lambda_4 + |\lambda_5| \qquad \text{if} \quad 0 < \lambda_4 < |\lambda_5| \tag{A.4b}$$

$$\sqrt{\lambda_1\lambda_2} > -\lambda_3 \qquad \text{if} \quad |\lambda_5| \leq \lambda_4 \tag{A.4c}$$

In our numerical integration of the renormalization group equations the initial values for the λ_i must be chosen to satisfy these stability constraints; these constraints should be respected by the fixed points as well. This is due to the fact that these are quartic constraints and if the potential is unstable at M_X with respect to these constraints, then it will be unstable at all lower scales. Moreover, should the potential develop a quartic instability at low energies the vacuum expectation values will grow until the stable minimum is found at a higher energy.

Spontaneous breaking of symmetry occurs when μ_1^2 and μ_2^2 are negative. The case of present interest is when both ϕ_1 and ϕ_2 develop vacuum expectation values. In this case the vacuum will conserve electric charge provided $\lambda_4 < 0$ and eq.(A.4a) is the relevant stability condition. Indeed, we seek a reason why electric charge conservation should be favored in these models. The vacuum expectation values for ϕ_1 and ϕ_2 are:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad , \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} \tag{A.5}$$

where v_1 and v_2 are real and $v_1{}^2 + v_2{}^2 = v^2 = (246\ \mathrm{Gev})^2$. Consistency requires $\lambda_5 < 0$ (explicitly one minimizes the potential allowing a relative phase between $\langle\phi_1\rangle$ and $\langle\phi_2\rangle$. One finds that $\langle\phi_1\rangle$ and $\langle\phi_2\rangle$ should be relatively real if $\lambda_5 < 0$).

It is easy to calculate the masses of the physical particles after symmetry breaking. There is a total of four: one charged ($H^\pm$), one neutral pseudoscalar (P), and two neutral scalars (H_1,H_2). The particles and their mass^2 are:

$$H^\pm = -\phi_1^\pm \sin\beta + \phi_2^\pm \cos\beta \quad , \quad m_\pm^2 = -\tfrac{1}{2}(\lambda_4+\lambda_5)v^2 \tag{A.6a}$$

$$\frac{1}{\sqrt{2}}P = -\mathrm{Im}\,\phi_1^0 \sin\beta + \mathrm{Im}\,\phi_2^0 \cos\beta \quad , \quad m_P^2 = -\lambda_5 v^2 \tag{A.6b}$$

$$\frac{1}{\sqrt{2}} H_1 = \mathrm{Re}\,\phi_1^0 \cos\alpha + \mathrm{Re}\,\phi_2^0 \sin\alpha \quad , \quad m_1^2 = \frac{1}{2}\eta_+ v^2 \tag{A.6c}$$

$$\frac{1}{\sqrt{2}} H_2 = -\mathrm{Re}\,\phi_1^0 \sin\alpha + \mathrm{Re}\,\phi_2^0 \cos\alpha \quad , \quad m_2^2 = \frac{1}{2}\eta_- v^2 \tag{A.6d}$$

where:

$$\tan\beta = \frac{v_2}{v_1} \tag{A.7}$$

$$\tan\alpha = \frac{\eta_+ - 2\lambda_1 \cos^2\beta}{(\lambda_3 + \lambda_4 + \lambda_5)\sin 2\beta} \tag{A.8}$$

and

$$\eta_\pm = (\lambda_1 \cos^2\beta + \lambda_2 \sin^2\beta) \pm \left[(\lambda_1 \cos^2\beta - \lambda_2 \sin^2\beta)^2 + (\lambda_3 + \lambda_4 + \lambda_5)^2 \sin^2 2\beta\right]^{1/2} \tag{A.9}$$

The massless Goldstone bosons that become the longitudinal components of W and Z are:

$$\begin{aligned} G^\pm &= \phi_1^\pm \cos\beta + \phi_2^\pm \sin\beta \\ \frac{1}{\sqrt{2}} G^0 &= \mathrm{Im}\,\phi_1^0 \cos\beta + \mathrm{Im}\,\phi_2^0 \sin\beta \end{aligned} \tag{A.10}$$

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Table I. Values of n_1, n_2 and n_3 for various cases considered.

	$N_H = 1$	$N_H = 2$
	$n_3 = 7.0$	$n_3 = 7.0$
$N = 3$	$n_2 = 3.1667$	$n_2 = 3.0$
	$n_1 = 6.8333$	$n_1 = 7.0$
	$n_3 = 5.6667$	$n_3 = 5.6667$
$N = 4$	$n_2 = 1.8333$	$n_2 = 1.6667$
	$n_1 = 9.0556$	$n_1 = 9.2222$
	$n_3 = 4.3333$	$n_3 = 4.3333$
$N = 5$	$n_2 = 0.5$	$n_2 = 0.3333$
	$n_2 = 11.2778$	$n_1 = 11.4444$

Table II. Values of the parameters a_k ($k=1,2,\ldots,11$) in the renormalization group equations of the Yukawa couplings for various cases.

	$N_H=1$	$N_H=2$									
		I	II	III	IV	V	VI	VII	VIII	IX	X
a_1	1	1	1	1	-	1	-	1	-	1	-
a_2	1	1	0	1	-	0	-	0	-	0	-
a_3	1	0	0	1	-	0	-	1	-	0	-
a_4	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	-	$-\frac{3}{2}$	-	-	-	-
a_5	1	1	0	1	0	-	0	-	-	-	-
a_6	1	1	1	1	1	-	1	-	-	-	-
a_7	1	0	1	1	0	-	1	-	-	-	-
a_8	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	0	$-\frac{3}{2}$	0	0	-	-
a_9	1	0	0	1	0	0	0	1	0	-	-
a_{10}	1	0	1	1	0	0	1	0	0	-	-
a_{11}	1	1	1	1	1	1	1	1	1	-	-

Table III. Values of the parameters α_i ($i=1,2,\ldots,23$) in the renormalization group equations of the scalar quartic couplings for various cases.

	$N_H=1$	$N_H=2$ I	II	III	IV	V	VI	VII	VIII	IX	X
α_1	1	0	0	1	0	0	0	1	0	1	0
α_2	1	0	1	1	0	0	1	0	0	0	0
α_3	1	1	1	1	1	1	1	1	1	0	0
α_4	1	0	0	1	0	0	0	1	0	1	0
α_5	1	0	1	1	0	0	1	0	0	0	0
α_6	1	1	1	1	1	1	1	1	1	0	0
α_7	-	1	1	0	0	1	0	0	0	0	0
α_8	-	1	0	0	1	0	0	0	0	0	0
α_9	-	0	0	0	0	0	0	0	0	0	0
α_{10}	-	1	1	0	0	1	0	0	0	0	0
α_{11}	-	1	0	0	1	0	0	0	0	0	0
α_{12}	-	0	0	0	0	0	0	0	0	0	0
α_{13}	-	1	1	1	0	1	0	1	0	1	0
α_{14}	-	1	1	1	1	0	1	0	0	0	0
α_{15}	-	1	1	1	1	1	1	1	1	0	0
α_{16}	-	1	0	0	1	0	0	0	0	0	0
α_{17}	-	1	1	1	0	1	0	1	0	1	0
α_{18}	-	1	1	1	1	0	1	0	0	0	0
α_{19}	-	1	1	1	1	1	1	1	1	0	0
α_{20}	-	1	0	0	1	0	0	0	0	0	0
α_{21}	-	1	1	1	0	1	0	1	0	1	0
α_{22}	-	1	1	1	1	0	1	0	0	0	0
α_{23}	-	1	1	1	1	1	1	1	1	0	0

Table IV. Fixed point values of the scalar quartic couplings in the two-Higgs model: Coupling Scheme I.

	m_t (GeV)	λ_1	λ_2	λ_3	λ_4	$\lvert\lambda_5\rvert$
	0	0.88	0.71	0.75	-1.11±0.03	0.062±0.046
N=4	50	0.84	0.71	0.73	-1.08±0.03	0.058±0.042
	172	0.69	0.77	0.45	-0.60	0.010±0.006
	0	0.60	0.47	0.51	-0.66	0.007±0.004
N=5	50	0.57	0.48	0.49	-0.64	0.007±0.004
	135	0.49	0.50	0.37	-0.46	0.003±0.002

Table V. Fixed point values of the scalar quartic couplings in the two-Higgs model: Coupling Scheme IV.

	m_t (GeV)	λ_1	λ_2	λ_3	λ_4	$\lvert\lambda_5\rvert$
	0	0.85	0.82	0.83	-1.30±0.03	0.076±0.042
N=4	50	0.81	0.83	0.82	-1.26±0.03	0.070±0.038
	172	0.68	0.90	0.50	-0.68±0.01	0.011±0.004
	0	0.59	0.57	0.58	-0.78	0.009±0.003
N=5	50	0.56	0.57	0.56	-0.76	0.008±0.003
	135	0.48	0.60	0.42	-0.53	0.003±0.001

Table VI. Fixed point masses (in GeV) of the physical Higgs particles: Coupling Scheme I.

	m_t	$m_\pm$	m_1	m_2	m_P
	0	188	190-231	0-105	"61"
N=4	50	186	188-225	0-105	"59"
	172	136	164-216	0-131	"25"
	0	142	144-191	0-106	"21"
N=5	50	140	143-186	0-105	"21"
	135	118	133-174	0-110	"13"

Table VII. Fixed point masses (in GeV) of the physical Higgs particles: Coupling Scheme IV.

	m_t	$m_\pm$	m_1	m_2	m_p
N=4	0	204	204-227	0-94	"68"
	50	201	201-224	0-97	"65"
	172	145	171-233	0-133	"26"
N=5	0	155	155-189	0-106	"23"
	50	152	153-186	0-104	"22"
	135	127	140-191	0-113	"13"

Table VIII. Range of the neutral scalar masses in the "semi-leptonic" coupling schemes (II and V) assuming m_t=0.

	II		V	
	N=4	N=5	N=4	N=5
m_1(GeV)	148-244	127-192	167-247	144-206
m_2(GeV)	0-146	0-127	0-165	0-142

Figure Captions

Fig. 1:
Flow of λ and g_t towards fixed points in the standard one Higgs doublet model. Open circles denote initial points. Crosses denote final fixed points.

Fig. 2:
Relation between the Higgs mass and the t-quark mass in the standard one Higgs model.

Fig. 3a:
Flow of λ_1 and λ_2 towards the fixed point. Coupling scheme I with four generations. The initial conditions on the gauge , Yukawa, and the other scalar quartic couplings are the same for all cases. Initial points of λ_1 and λ_2 are indicated by open circles. The final fixed point is indicated by a cross. Solid circles along the curve label energy steps (see text for details) to demonstrate how fast the fixed point is reached. Fixed points for the λ_i are λ_1=0.66, λ_2=0.75, λ_3=0.45 λ_4=-0.57, λ_5=0.02.

Fig. 3b:
Flow of λ_1 and λ_3 towards the fixed point. The initial conditions are the same as in Fig. 3a. Fixed points for the λ_i are the same as in Fig. 3a except that $\lambda_5 \longrightarrow$ 0.02 - 0.04.

Fig. 3c:
Flow of λ_1 and λ_4 towards the fixed point. Initial conditions are the same as in Fig.3a. Fixed points for the λ_i remain the same as in Fig. 3a,except that $\lambda_5 \longrightarrow$ 0.02 - 0.07.

Fig. 4a:
Masses of the neutral scalar Higgs particles ($m_{1,2}$) as a function of $(v_2/v_1)^2$ in coupling scheme I. Solid lines denote N = 4; dashed lines denote N = 5. Mass of the t-quark is assumed to be small (less than 50 Gev).

Fig. 4b:
Same as in Fig. 4a, but for coupling scheme IV.

Fig. 5:
$\lambda_1 \cdots \lambda_5$ are plotted vs. log (energy in Gev) in the leptonic coupling scheme X to demonstrate how the singularity occurs for a specific case. The dotted line indicates the energy scale at which the λ_i become unstable, which is $\sim 10^{12.75}$ Gev. The singularity occurs only when we reach $\sim 10^7$ Gev.

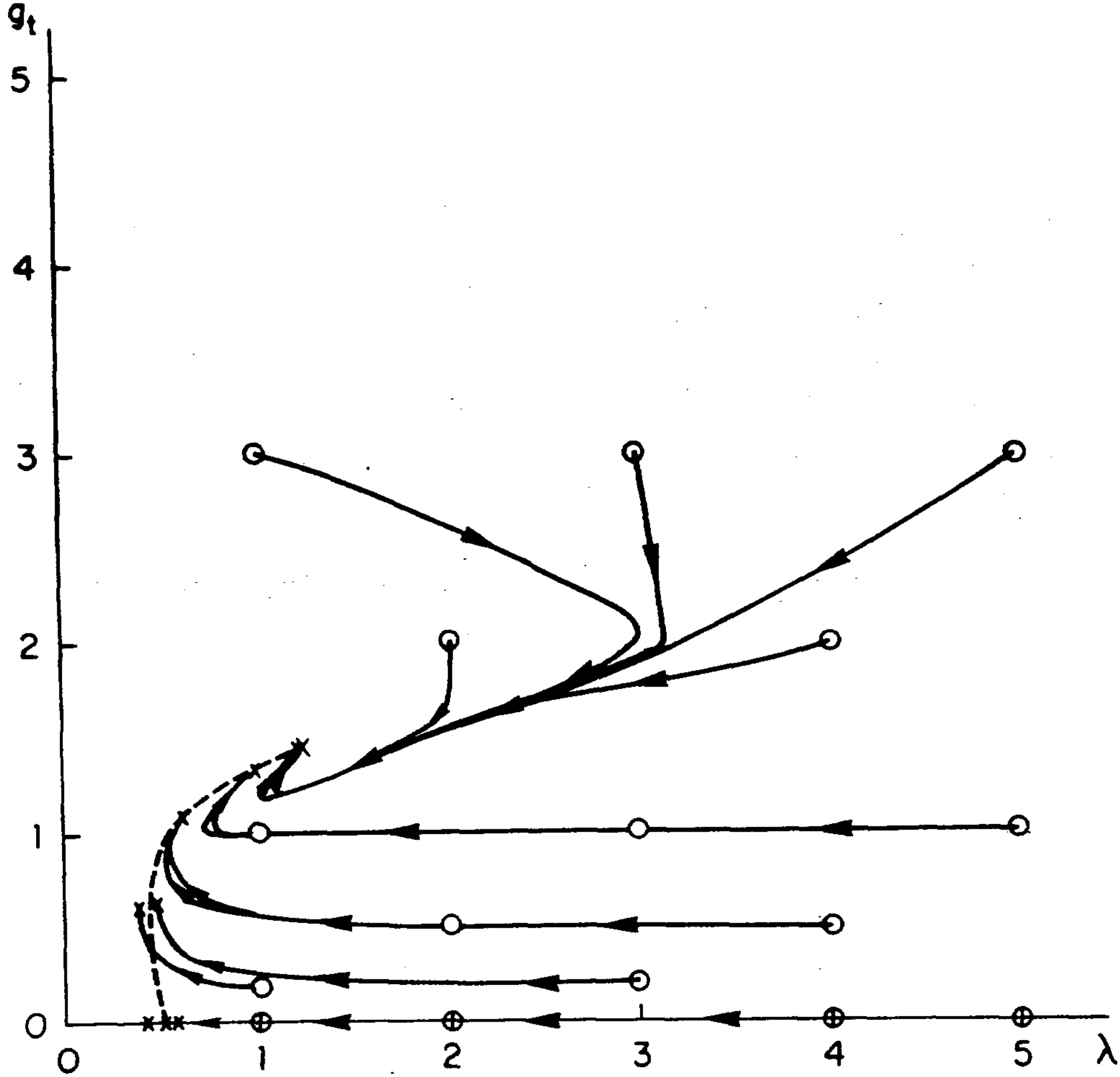

Fig. 1

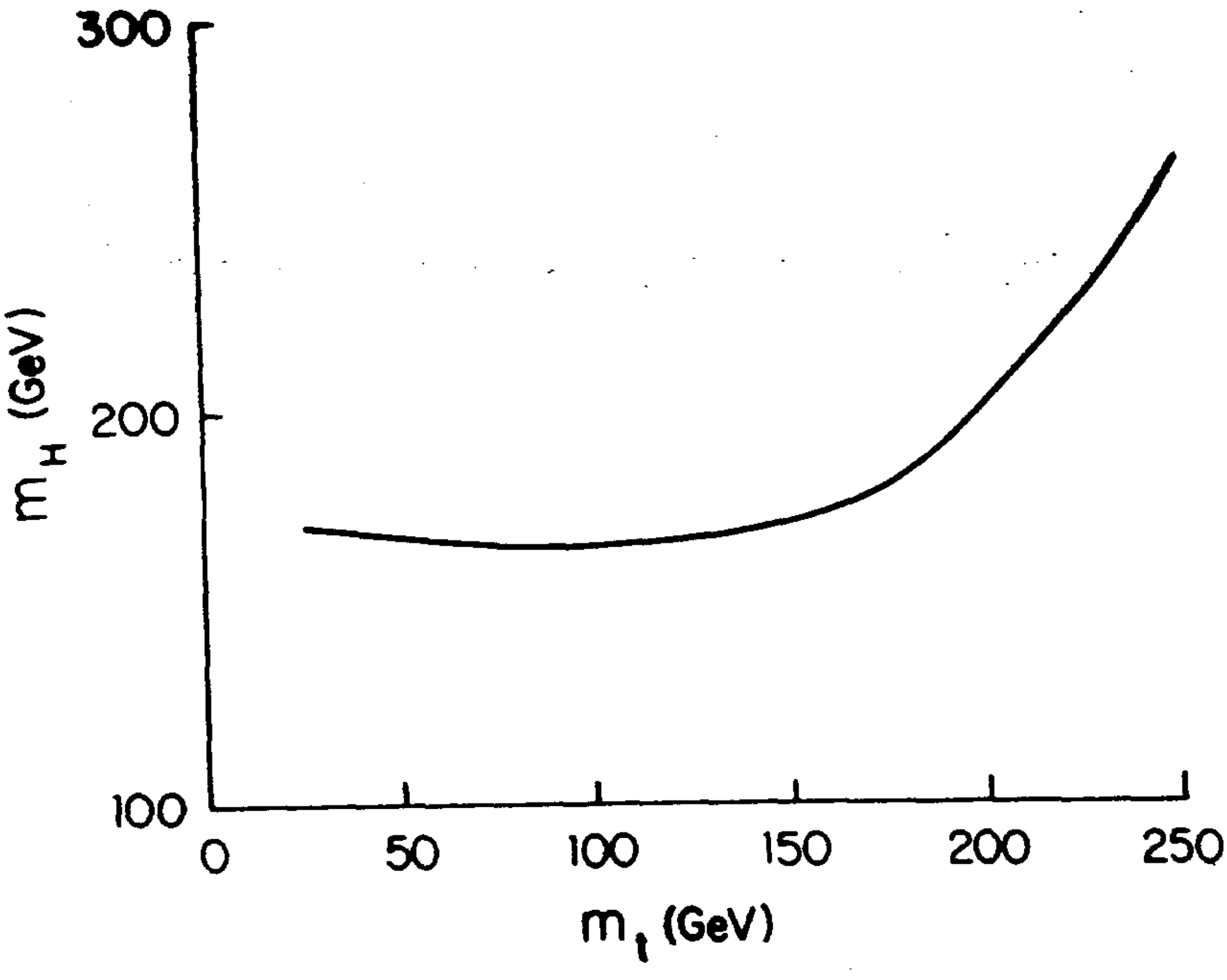

Fig. 2

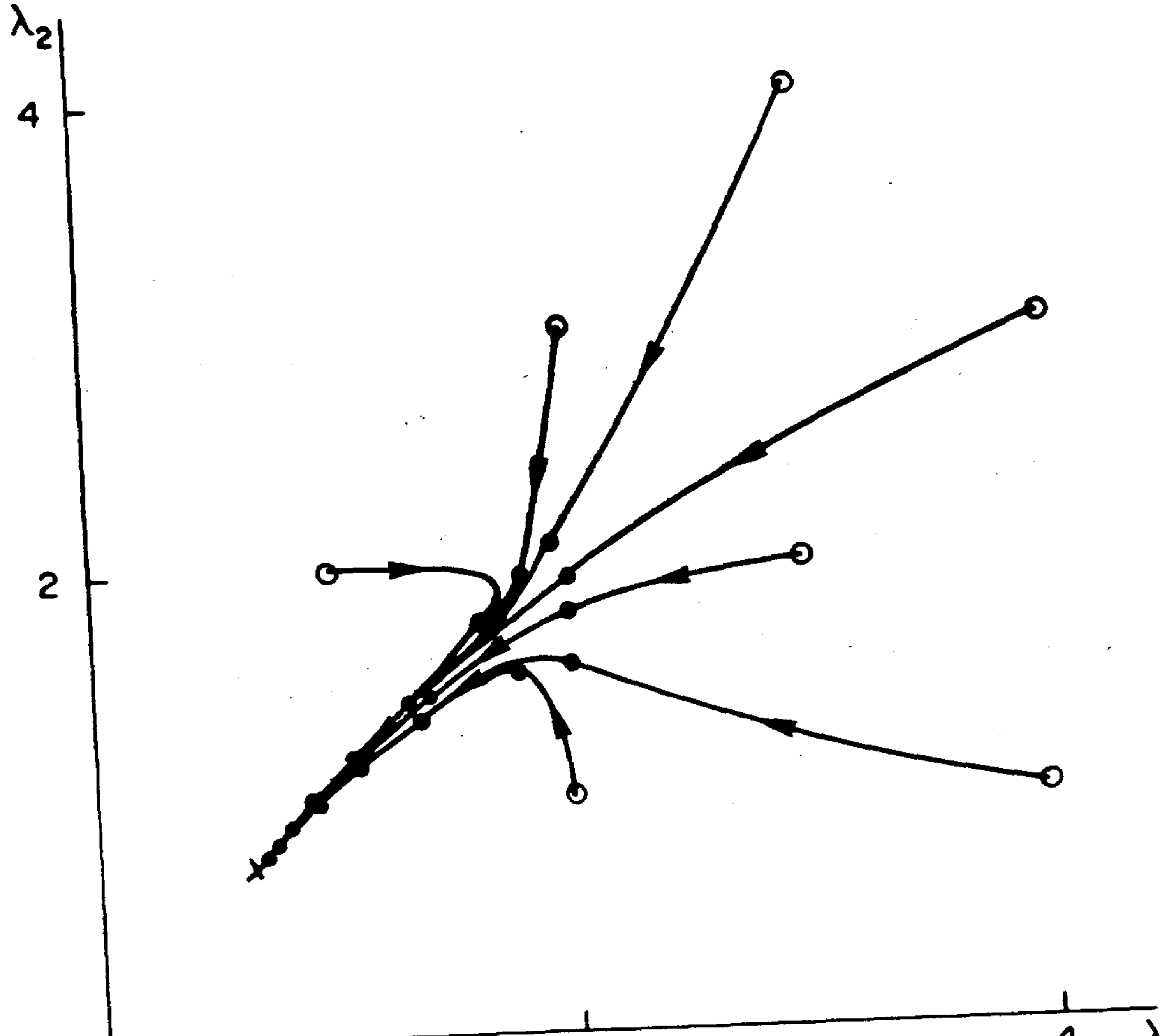

Fig. 3a

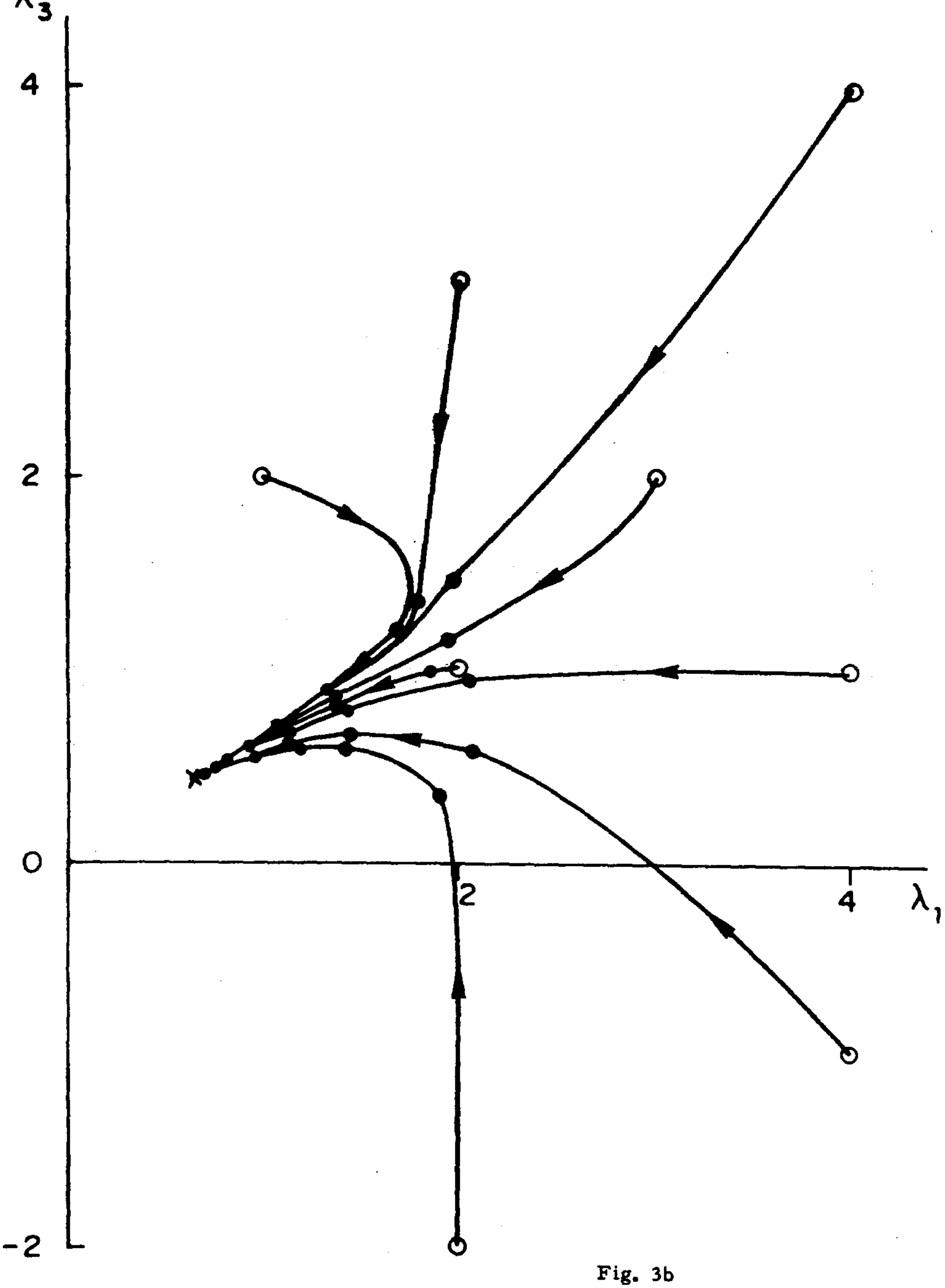

Fig. 3b

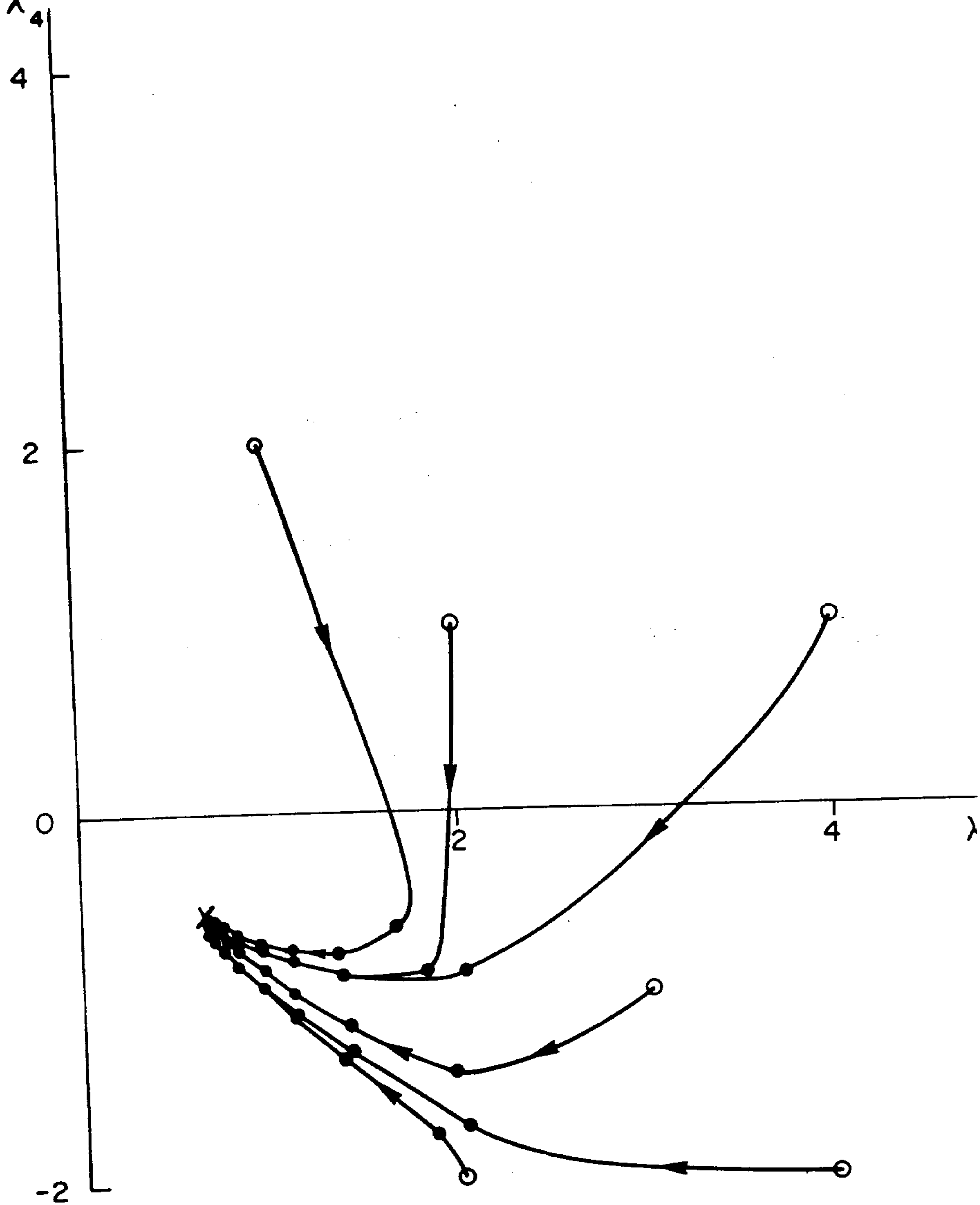

Fig. 3c

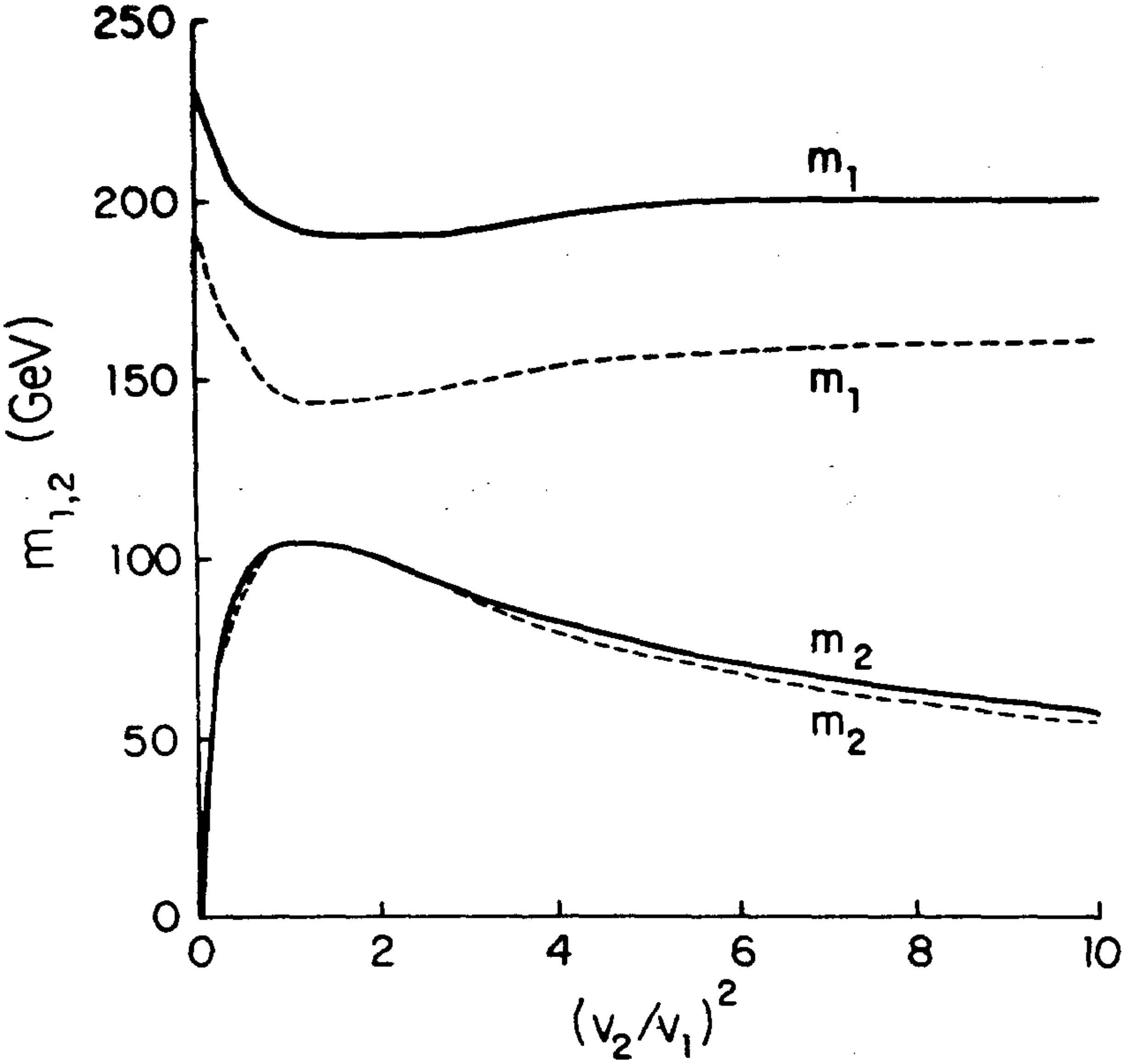

Fig. 4a

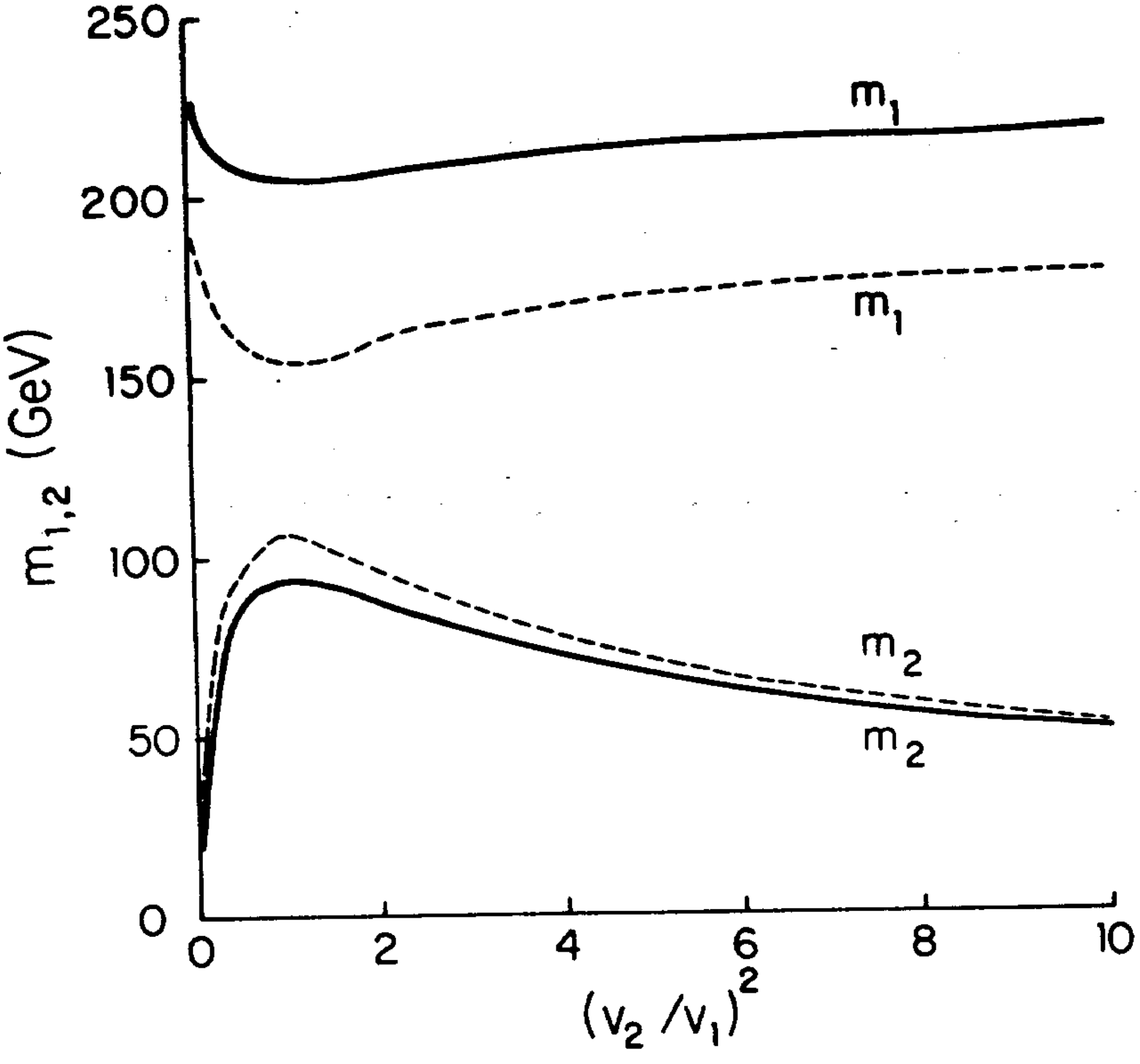

Fig. 4b

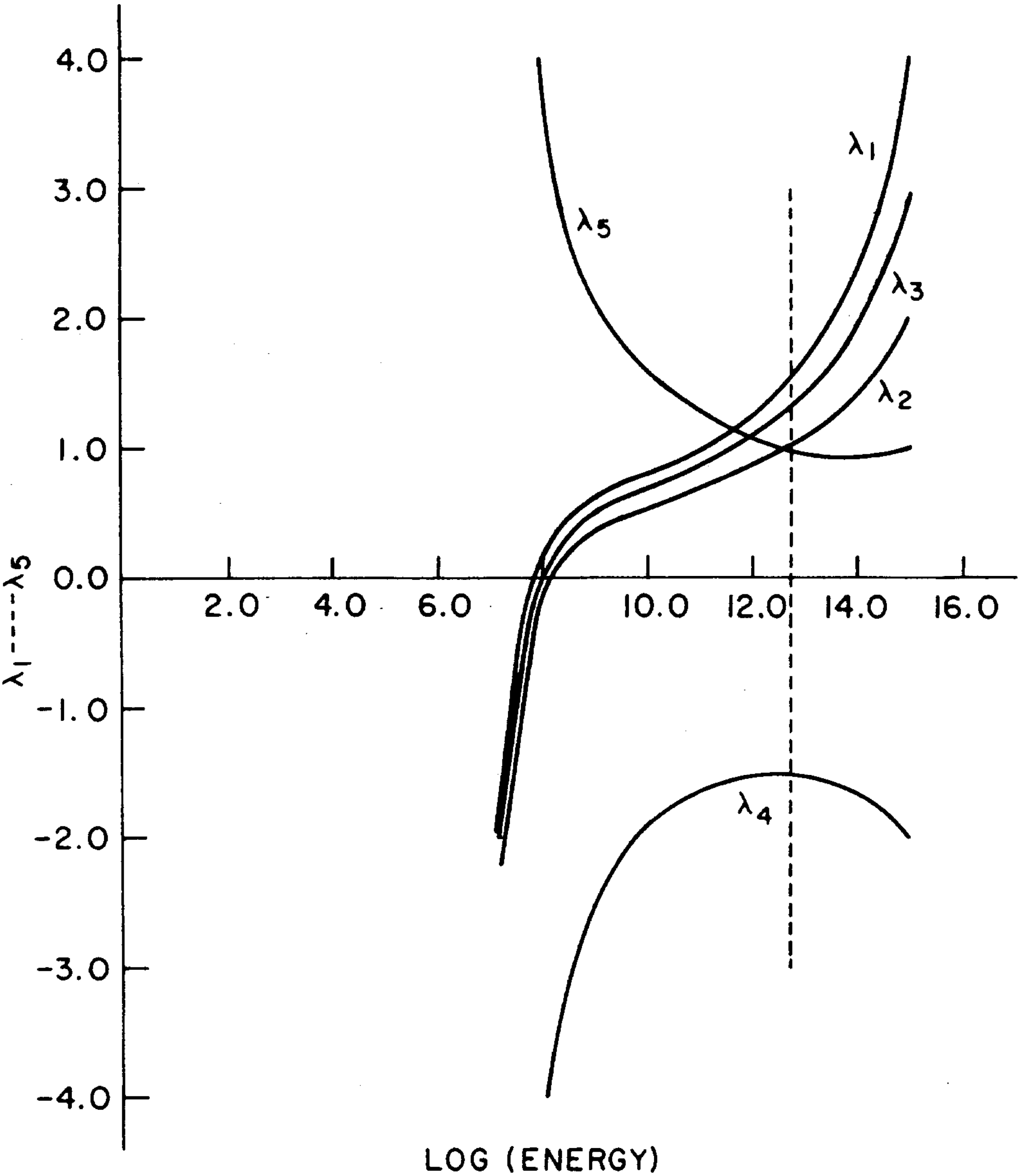
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
λ_1---λ_5
2.0
4.0
6.0
10.0
12.0
14.0
16.0
λ_1
λ_5
λ_3
λ_2
λ_4
LOG (ENERGY)

Fig. 5